Changes in subjective well-being with retirement: assessing savings adequacy*

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Appendix

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Trivariate Ordered Probit Response Model

The system is composed of three subjective well-being measures. Let y_1^* denote the standard of living, y_2^* financial security and y_3^* overall happiness. Each well-being measure is determined by a latent index which is a function of individual determinants $\mathbf{x}_i'$ and an idiosyncratic error term e_{ji} :

$$\begin{aligned} y_{1i}^* &= \mathbf{x}_i'\beta_1 + e_{1i} \\ y_{2i}^* &= \mathbf{x}_i'\beta_2 + e_{2i} \\ y_{3i}^* &= \mathbf{x}_i'\beta_3 + e_{3i} \end{aligned} \tag{A.1}$$

where β_j for $j = \{1, 2, 3\}$ are parameters to be estimated. The actual y_{ji}^* is not observed, individuals report y_{ji} which indicates that y_{1i}^* falls into one of 3 rank ordered categories $y_{ji}^* \in \{worse \prec same \prec better\}$ or, without loss generality, $y_{ji} \in \{1, 2, 3\}$. Hence the set of observed ordered responses are:

$$\begin{aligned} y_{1i} &= \begin{cases} 1 \text{ if } y_{1i}^* \leq \mu_{11} \\ 2 \text{ if } \mu_{11} < y_{1i}^* \leq \mu_{12} \\ 3 \text{ if } \mu_{12} < y_{1i}^* \end{cases} \\ y_{2i} &= \begin{cases} 1 \text{ if } y_{2i}^* \leq \mu_{21} \\ 2 \text{ if } \mu_{21} < y_{2i}^* \leq \mu_{22} \\ 3 \text{ if } \mu_{22} < y_{2i}^* \end{cases} \\ y_{3i} &= \begin{cases} 1 \text{ if } y_{3i}^* \leq \mu_{31} \\ 2 \text{ if } \mu_{31} < y_{3i}^* \leq \mu_{32} \\ 3 \text{ if } \mu_{13} < y_{3i}^* \end{cases} \end{aligned} \tag{A.2}$$

where $\{\mu_{j1}, \mu_{j2}\}$ partition the y_j^* scale into three segments.

It is assumed the idiosyncratic error terms e_{ji} are drawn from a joint standard normal distribution. The three outcomes form a trivariate ordered probit system with latent error structure $\mathbf{e} \sim \mathbf{N}(0, \Sigma)$ where $\Sigma = \begin{bmatrix} 1 & & \\ \rho_{12} & 1 & \\ \rho_{13} & \rho_{23} & 1 \end{bmatrix}$.

The log-likelihood function for this model is composed of the following 27 distinct branches:

$$\begin{aligned} \Pr(y_{1i} = 1, y_{2i} = 1, y_{3i} = 1) &= \int_{-\infty}^{\mu_{31} - \mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21} - \mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11} - \mathbf{x}_i'\beta_1} \phi(\tau_1, \tau_2, \tau_3; \Sigma)\, d\tau_1 d\tau_2 d\tau_3 \\ &= \Phi_T(\mu_{11} - \mathbf{x}_i'\beta_1, \mu_{21} - \mathbf{x}_i'\beta_2, \mu_{31} - \mathbf{x}_i'\beta_3; \Sigma) \end{aligned}$$

$$\begin{aligned} \Pr(y_{1i} = 1, y_{2i} = 1, y_{3i} = 2) &= \int_{\mu_{31} - \mathbf{x}_i'\beta_3}^{\mu_{32} - \mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21} - \mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11} - \mathbf{x}_i'\beta_1} \phi(\tau_1, \tau_2, \tau_3; \Sigma)\, d\tau_1 d\tau_2 d\tau_3 \\ &= \int_{-\infty}^{\mu_{32} - \mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21} - \mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11} - \mathbf{x}_i'\beta_1} \phi(\tau_1, \tau_2, \tau_3; \Sigma)\, d\tau_1 d\tau_2 d\tau_3 \\ &\quad - \int_{-\infty}^{\mu_{31} - \mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21} - \mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11} - \mathbf{x}_i'\beta_1} \phi(\tau_1, \tau_2, \tau_3; \Sigma)\, d\tau_1 d\tau_2 d\tau_3 \\ &= \Phi_T(\mu_{11} - \mathbf{x}_i'\beta_1, \mu_{21} - \mathbf{x}_i'\beta_2, \mu_{32} - \mathbf{x}_i'\beta_3; \Sigma) \\ &\quad - \Phi_T(\mu_{11} - \mathbf{x}_i'\beta_1, \mu_{21} - \mathbf{x}_i'\beta_2, \mu_{31} - \mathbf{x}_i'\beta_3; \Sigma) \end{aligned}$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1,y_{2i}=1,y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{21}\right)d\tau_1 d\tau_2 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2;\rho_{21}\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1,y_{2i}=2,y_{3i}=1\right) &= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1,y_{2i}=2,y_{3i}=2\right) &= \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad +\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1,y_{2i}=2,y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3
\end{aligned}
$$

$$
\begin{aligned}
& - \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2; \rho_{12}\right) d\tau_1 d\tau_2 \\
& + \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2; \rho_{12}\right) \\
& - \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
& - \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2; \rho_{12}\right) \\
& + \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1, y_{2i}=3, y_{3i}=1\right) = \; & \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_3; \rho_{13}\right) d\tau_1 d\tau_3 \\
& - \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{31}-\mathbf{x}_i'\beta_3; \rho_{13}\right) \\
& - \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1, y_{2i}=3, y_{3i}=2\right) = \; & \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_3; \rho_{13}\right) d\tau_1 d\tau_3 \\
& - \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
& - \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_3; \rho_{13}\right) d\tau_1 d\tau_3 \\
& + \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{32}-\mathbf{x}_i'\beta_3; \rho_{13}\right) \\
& - \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
& - \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{31}-\mathbf{x}_i'\beta_3; \rho_{13}\right) \\
& + \Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=1, y_{2i}=3, y_{3i}=3\right) = \; & \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty} \int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1\right) d\tau_1 - \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2; \rho_{12}\right) d\tau_1 d\tau_2 \\
& - \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_3; \rho_{13}\right) d\tau_1 d\tau_3
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
= \;& \Phi\left(\mu_{11}-\mathbf{x}_i'\beta_1\right)-\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2;\rho_{12}\right) \\
&-\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&+\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2,y_{2i}=1,y_{3i}=1\right) &= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2,y_{2i}=1,y_{3i}=2\right) &= \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad +\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2,y_{2i}=1,y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2 \\
&\quad -\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2\rho_{12}\right)d\tau_1 d\tau_2 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3
\end{aligned}
$$

$$
\begin{aligned}
&= \Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2; \rho_{12}\right) - \Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2; \rho_{12}\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&\quad +\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2, y_{2i}=2, y_{3i}=1\right) &= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&\quad +\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2, y_{2i}=2, y_{3i}=2\right) &= \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
&+\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right)\\
&-\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)\\
&+\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)\\
&+\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)\\
&-\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2, y_{2i}=2, y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&= \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2\\
&\quad-\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2\\
&\quad-\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2\\
&\quad+\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2\\
&\quad-\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&\quad+\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&\quad+\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&\quad-\int_{-\infty}^{32}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&= \Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2;\rho_{12}\right)\\
&\quad-\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2;\rho_{12}\right)\\
&\quad-\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2;\rho_{12}\right)\\
&\quad+\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2;\rho_{12}\right)\\
&\quad-\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)\\
&\quad+\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)\\
&\quad+\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)\\
&\quad-\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2, y_{2i}=3, y_{3i}=1\right) &= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3\\
&\quad-\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&\quad-\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
= \;& \Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&-\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&-\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&+\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2,y_{2i}=3,y_{3i}=2\right) &= \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad -\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad +\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad +\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad +\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=2,y_{2i}=3,y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty}\int_{\mu_{11}-\mathbf{x}_i'\beta_1}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1\right)d\tau_1 - \int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1\right)d\tau_1 \\
&\quad -\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2\\
&-\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{13}\right)d\tau_1 d\tau_3\\
&+\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&+\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3\\
&-\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
=\;&\Phi\left(\mu_{12}-\mathbf{x}_i'\beta_1\right)-\Phi\left(\mu_{11}-\mathbf{x}_i'\beta_1\right)\\
&-\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2;\rho_{12}\right)\\
&+\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2;\rho_{12}\right)\\
&-\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{13}\right)\\
&+\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)\\
&+\Phi_B\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{13}\right)\\
&-\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=1,y_{3i}=1\right)&=\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&=\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3\\
&\quad-\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&=\Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{23}\right)\\
&\quad-\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=1,y_{3i}=2\right)&=\int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&=\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3\\
&\quad-\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3\\
&\quad-\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&\quad+\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3\\
&=\Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{23}\right)\\
&\quad-\Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{23}\right)\\
&\quad-\Phi_T\left(\mu_{11}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)\\
&\quad+\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=1,y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2\right)d\tau_2 \\
&\quad -\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{11}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi\left(\mu_{21}-\mathbf{x}_i'\beta_2\right) \\
&\quad -\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2;\rho_{12}\right) \\
&\quad -\Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{23}\right) \\
&\quad +\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=2,y_{3i}=1\right) &= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{23}\right) \\
&\quad -\Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{23}\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{21}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=2,y_{3i}=2\right) &= \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3
\end{aligned}
$$

$$
\begin{aligned}
&- \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2, \tau_3; \rho_{23}\right) d\tau_2 d\tau_3 \\
&+ \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2, \tau_3; \rho_{23}\right) d\tau_2 d\tau_3 \\
&+ \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
&- \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \;& \Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \rho_{23}\right) \\
&- \Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \rho_{23}\right) \\
&- \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&+ \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&- \Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \rho_{23}\right) \\
&+ \Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \rho_{23}\right) \\
&+ \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&- \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{31}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3, y_{2i}=2, y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty} \int_{\mu_{21}-\mathbf{x}_i'\beta_2}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2\right) d\tau_2 - \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2\right) d\tau_2 \\
&- \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2; \rho_{12}\right) d\tau_1 d\tau_2 \\
&+ \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2; \rho_{12}\right) d\tau_1 d\tau_2 \\
&- \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2, \tau_3; \rho_{23}\right) d\tau_2 d\tau_3 \\
&+ \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2, \tau_3; \rho_{23}\right) d\tau_2 d\tau_3 \\
&+ \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
&- \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{21}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_3; \Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi\left(\mu_{22}-\mathbf{x}_i'\beta_2\right) - \Phi(\mu_{21}-\mathbf{x}_i'\beta_2) \\
&- \Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2; \rho_{12}\right) \\
&+ \Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2; \rho_{12}\right) \\
&- \Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \rho_{23}\right) \\
&+ \Phi_B\left(\mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \rho_{23}\right) \\
&+ \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{22}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right) \\
&- \Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1, \mu_{21}-\mathbf{x}_i'\beta_2, \mu_{32}-\mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=3,y_{3i}=1\right) &= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\phi\left(\tau_3\right)d\tau_3 - \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{13}\right)d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi\left(\mu_{31}-\mathbf{x}_i'\beta_3\right)-\Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{23}\right) \\
&\quad -\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad +\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=3,y_{3i}=2\right) &= \int_{\mu_{31}-\mathbf{x}_i'\beta_3}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\phi\left(\tau_3\right)d\tau_3 - \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\phi\left(\tau_3\right)d\tau_3 + \int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2,\tau_3;\rho_{23}\right)d\tau_2 d\tau_3 \\
&\quad +\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_3;\rho_{13}\right)d\tau_1 d\tau_3 \\
&\quad -\int_{-\infty}^{\mu_{31}-\mathbf{x}_i'\beta_3}\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2,\tau_{3;}\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= \Phi\left(\mu_{32}-\mathbf{x}_i'\beta_3\right)-\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad -\Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\rho_{23}\right) \\
&\quad +\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{32}-\mathbf{x}_i'\beta_3;\Sigma\right) \\
&\quad -\Phi\left(\mu_{31}-\mathbf{x}_i'\beta_3\right)+\Phi_B\left(\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{23}\right) \\
&\quad +\Phi_B\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{31}-\mathbf{x}_i'\beta_3;\rho_{13}\right) \\
&\quad -\Phi_T\left(\mu_{12}-\mathbf{x}_i'\beta_1,\mu_{22}-\mathbf{x}_i'\beta_2,\mu_{31}-\mathbf{x}_i'\beta_3;\Sigma\right)
\end{aligned}
$$

$$
\begin{aligned}
\Pr\left(y_{1i}=3,y_{2i}=3,y_{3i}=3\right) &= \int_{\mu_{32}-\mathbf{x}_i'\beta_3}^{\infty}\int_{\mu_{22}-\mathbf{x}_i'\beta_2}^{\infty}\int_{\mu_{12}-\mathbf{x}_i'\beta_1}^{\infty}\phi\left(\tau_1,\tau_2,\tau_3;\Sigma\right)d\tau_1 d\tau_2 d\tau_3 \\
&= 1-\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\phi\left(\tau_2\right)d\tau_2 - \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1\right)d\tau_1 \\
&\quad +\int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2}\int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1}\phi\left(\tau_1,\tau_2;\rho_{12}\right)d\tau_1 d\tau_2
\end{aligned}
$$

$$
\begin{aligned}
& -\int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \phi\left(\tau_3\right) d\tau_3 + \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \phi\left(\tau_2, \tau_3; \rho_{23}\right) d\tau_2 d\tau_3 \\
& + \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_3; \rho_{13}\right) d\tau_1 d\tau_3 \\
& - \int_{-\infty}^{\mu_{32}-\mathbf{x}_i'\beta_3} \int_{-\infty}^{\mu_{22}-\mathbf{x}_i'\beta_2} \int_{-\infty}^{\mu_{12}-\mathbf{x}_i'\beta_1} \phi\left(\tau_1, \tau_2, \tau_{3;}\Sigma\right) d\tau_1 d\tau_2 d\tau_3 \\
= \; & 1 - \Phi\left(\mu_{22} - \mathbf{x}_i'\beta_2\right) - \Phi(\mu_{12} - \mathbf{x}_i'\beta_1) \\
& + \Phi_B\left(\mu_{12} - \mathbf{x}_i'\beta_1, \mu_{22} - \mathbf{x}_i'\beta_2; \rho_{12}\right) \\
& - \Phi\left(\mu_{32} - \mathbf{x}_i'\beta_3\right) + \Phi_B\left(\mu_{22} - \mathbf{x}_i'\beta_2, \mu_{32} - \mathbf{x}_i'\beta_3; \rho_{23}\right) \\
& + \Phi_B\left(\mu_{12} - \mathbf{x}_i'\beta_1, \mu_{32} - \mathbf{x}_i'\beta_3; \rho_{13}\right) \\
& - \Phi_T\left(\mu_{12} - \mathbf{x}_i'\beta_1, \mu_{22} - \mathbf{x}_i'\beta_2, \mu_{32} - \mathbf{x}_i'\beta_3; \Sigma\right)
\end{aligned}
$$

where $\Phi(.), \Phi_B(.,.;.)$ and $\Phi_T(.,.,.;.)$ denote the standard normal univariate, bivariate and trivariate CDF, respectively, and $\phi(.)$ denotes the standard normal PDF.

The log-likelihood function for the sample is given by

$$
\sum_{i=1}^{n} \sum_{l=1}^{3} \sum_{k=1}^{3} \sum_{j=1}^{3} d_{ijkl} \ln \Pr(y_{1i} = j, y_{2i} = k, y_{3i} = l) \tag{A.3}
$$

where d_{ijkl} is an indicator variable defined by $\mathbf{1}\left(y_{1i} = j, y_{2i} = k, y_{3i} = l\right)$ where $\mathbf{1}\left(A\right)$ is the indicator function which is equal to 1 when condition A is true, and is 0 otherwise. The system is estimated using Full Information Maximum Likelihood methods.